$$\frac{dP}{dt} = -m(P-P_c)P$$

CARRYING CAPACITY

AS $t \to \infty$, $P \to P_c$

BUT

IF P_0 IS SMALL,

$P \to P_c$

(NOTE: $P \to \infty$ AT FINITE VALUE OF $t<0$)

$$\frac{dP}{dt} = m(P-P_s)P$$

SURVIVAL THRESHOLD

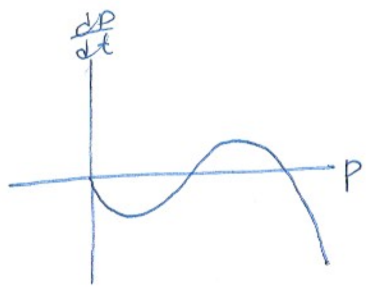
IF P_0 IS SMALL,

$P \to 0$

$P \to \infty$ AT FINITE VALUE OF $t>0$

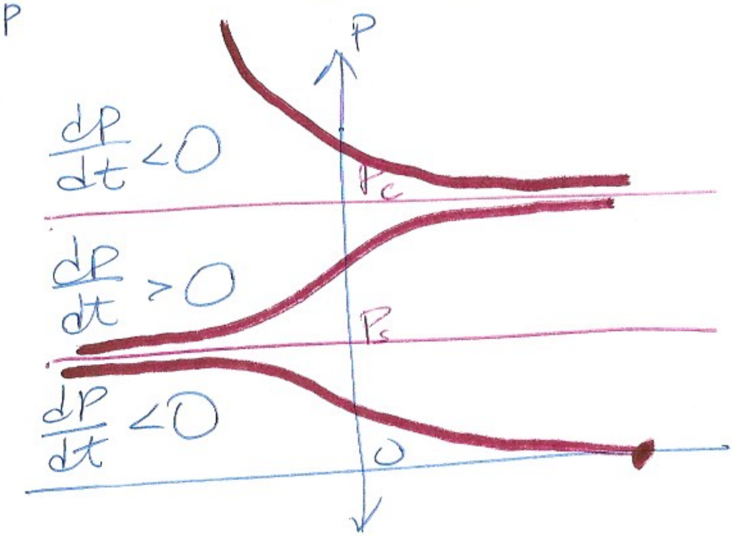
$$\frac{dP}{dt} = -m(P-P_c)(P-P_s)P \; ; \quad m, P_c, P_s > 0$$

$P_s < P_c$

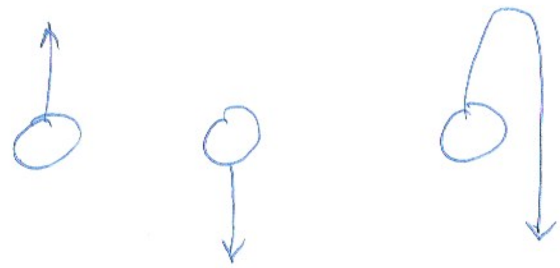


EQ SOL'N

$-m(P-P_c)(P-P_s)P = 0 \to P = 0, P_s, P_c$



RISING + FALLING OBJECTS



NEWTON'S LAW

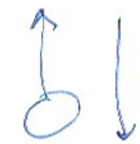
$$\vec{F} = m\vec{a}, \quad m > 0$$

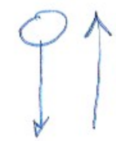
$$\vec{F} = \vec{F_g} + \vec{F_a}$$

$$F = F_g + F_a$$

$F_g \downarrow mg$

$g = 9.8\ m/s^2$
or
$32\ ft/s^2$





$$\|\vec{F_a}\| \propto \|\vec{v}\|$$

$$|F_a| = k|v|, \quad k > 0$$

LET $v > 0 \longrightarrow$ OBJECT MOVING UPWARD

$a > 0 \longrightarrow$ UPWARD ACCELERATION

$F > 0 \longrightarrow$ UPWARD FORCE

IF $v > 0$, OBJECT $\uparrow$, $F_g \downarrow$, $F_a \downarrow$

$F_g < 0$, $F_a < 0 \longrightarrow$

$$ma = -mg - kv$$

$$m\frac{dv}{dt} = -mg - kv$$

$$\frac{dv}{dt} = -g - \frac{k}{m}v$$

IF $v < 0$, OBJECT $\downarrow$, $F_g \downarrow$, $F_a \uparrow$

$F_g < 0$, $F_a > 0 \longrightarrow$

$$ma = -mg - kv$$

$$\frac{dv}{dt} = -g - \frac{k}{m}v$$

($v>0$ UPWARD)

$$\frac{dv}{dt} = -g - \frac{k}{m}v \;, \quad v(0) = v_0$$

$$\frac{dv}{dt} + \frac{k}{m}v = -g \qquad \mu = e^{\int \frac{k}{m}dt} = e^{\frac{k}{m}t}$$

$$e^{\frac{k}{m}t}\frac{dv}{dt} + \frac{k}{m}e^{\frac{k}{m}t}v = -ge^{\frac{k}{m}t} \qquad \frac{d}{dt}e^{\frac{k}{m}t} = \frac{k}{m}e^{\frac{k}{m}t} \checkmark$$ CHECKPOINT

$$e^{\frac{k}{m}t}v = \int -ge^{\frac{k}{m}t}dt + C$$

$$e^{\frac{k}{m}t}v = -\frac{gm}{k}e^{\frac{k}{m}t} + C$$

$$v = -\frac{gm}{k} + Ce^{-\frac{k}{m}t}$$

$$v_0 = -\frac{gm}{k} + C$$

$$C = v_0 + \frac{mg}{k}$$

$$v(t) = -\frac{mg}{k} + \left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t}$$

$$= \frac{mg}{k}\left(e^{-\frac{k}{m}t} - 1\right) + v_0\left(e^{-\frac{k}{m}t}\right) \longleftarrow$$

AT $t=0$ $\frac{mg}{k}(0) + v_0($

AS $t\to\infty$ $\frac{mg}{k}(-1) + v_0(0$

$$\lim_{t\to\infty} v(t) = -\frac{mg}{k} < 0 \qquad \Downarrow \quad |\text{TERMINAL VELOCITY}| = \frac{mg}{k}$$

AT WHAT TIME DOES THE OBJECT REACH ITS PEAK?

IE, $V(t_{PEAK}) = 0$

$$v(t) = -\frac{mg}{k} + \left(V_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} = 0 \quad @ \quad t = t_{PEAK}$$

$$\frac{k}{mg}\cancel{e^{\frac{k}{m}t}}\left(V_0 + \frac{mg}{k}\right)\cancel{e^{-\frac{k}{m}t}} = \frac{\cancel{mg}}{\cancel{k}}\, e^{\frac{k}{m}t}\,\frac{\cancel{k}}{\cancel{mg}}$$

$$\frac{kV_0}{mg} + 1 = e^{\frac{k}{m}t}$$

$$\ln\left(\frac{kV_0}{mg} + 1\right) = \frac{k}{m}t$$

$$t_{PEAK} = \frac{m}{k}\ln\left(\frac{kV_0}{mg} + 1\right)$$

AS g INCREASES, t_{PEAK} DECREASES CONFIRMED $\frac{kV_0}{mg}$ DECR

$\frac{m}{k}\ln\left(\frac{kV_0}{mg} + 1\right)$ DECR

AS V_0 INCREASES, t_{PEAK} INCREASES CONFIRMED $\frac{kV_0}{mg}$ INCR

$\frac{m}{k}\ln\left(\frac{kV_0}{mg} + 1\right)$ INCR

AS m INCREASES, NEED TO LOOK @ $\frac{dt_{PEAK}}{dm}$

AS k INCREASES, NEED TO LOOK @ $\frac{dt_{PEAK}}{dk}$

DO IT YOURSELF